# Rain drop size distribution (DSD) in Tropical Region of Akure South-Western Nigeria Using Micro Rain Radar Data 

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#### Abstract

Rain drop size distribution and some associated rain parameters such as the rain rates (R), liquid water content (M), and the falling velocities (W) as observed using a vertically pointing Micro Rain Radar (MRR) at the Department of Physics, the Federal University of Technology Akure ( $7015^{\prime} \mathrm{N}, 5015^{\prime} \mathrm{E}$ ), a tropical location in Nigeria were analyzed in this study. The parameters were measured from the ground level to a height of 4.8 km above sea level with a vertical resolution of 0.16 km and over a total of 30 range gates with 1-minute integration time. The measurements covered a period of four years (2008-2011). The study established relationships between all the parameters and the results shows typical values for negative exponential rainfall drop size distribution (DSD) similar to that of Marshall-Palmer for both stratiform and convective rain. At 0.01\% of time, the measured rain rate was underestimated by $35 \%$ when compared with the ITU-R recommendation for this region and it was observed that over $85 \%$ of the total rainfall in this part of the world is stratiform while the remaining $15 \%$ are convective except for the month of October which is the peak of the rainy season in the year where a high number of convective rain is observed. The results of this study may assist to improve the design and planning of terrestrial and satellite radio communication system in this location.


Keywords: Micro rain radar, Rain microstructure, Drop size distribution, Stratiform and Convective.

### 1.0 Introduction

Signal impairment due to rain (attenuation) is the most important factor at the millimetre wave frequencies and it is the limiting factor in satellite/terrestrial link design especially for tropical regions that experience very high amount of rainfall (Jassal et al. 2011). Demands for allocation for higher ends of electromagnetic spectrum is increasing daily in order to meet the demand of higher data rate for various communication and multimedia requirements. Hence, access to higher frequencies up to Super High Frequency (SHF) and Extremely High Frequency (EHF) are now being proposed for these satellite services. SHF and EHF can cope very well with higher data rates than the current Microwave system this is because the services are relatively free of congestion. However, attenuation that occurs due to atmospheric gasses, hales, clouds and rain increases significantly as frequency increases. It is important to note that the largest attenuation being experienced by communication signal is due to rain, (Adrian, 2011).

In the study of rainfall, the most important parameter to estimate with regards to rain drop size distribution (DSD) is the rain rate (R) (Ochou et al. 2011). It is one of the parameters measured from the ground level to the height of 4.8 km using vertically pointing micro rain radar (MRR).

Various researchers which include Marshall and Palmer, Joss and Waldvogel, Joss and Gori etc. had worked on modelling of rain drop size distribution. Exponential DSD model otherwise referred to as the Marshall and Palmer (1948) has been the most widely used analytical parameterization for the rain drop size distribution. It follows a function of the form:

$$
\begin{equation*}
N(D)=N_{o} \exp (-\Lambda D) \tag{1}
\end{equation*}
$$

where $\mathrm{N}(\mathrm{D})$ is the concentration of raindrops per diameter in the diameter interval dD in mm , $D$ is the rain drop diameter, $N_{o}$ is the intercept parameter with fixed value of $8 \times 10^{3} \mathrm{~mm}^{-1} \mathrm{~m}^{-3}$, $\wedge\left(\mathrm{mm}^{-1}\right)$ is the slope parameter and is defined as,

$$
\begin{equation*}
\wedge=4.1 \mathrm{R}^{-0.21} \mathrm{~mm}^{-1} \tag{2}
\end{equation*}
$$

where $R$ is the rainfall rate ( $\mathrm{mm} / \mathrm{h}$ ).
Marshall-Palmer discovered that rain DSDs’ for several rain rates, the exponential function does not fit the observation. Hence, it is sometimes necessary to consider the Marshall Palmer curves applicable at diameters greater than 1-1.5mm. (Battan, 1973).

## 2. Data Collection

The major equipment used for this work is the Micro Rain Radar which is an frequency modulated continuous wave (FM-CW) Doppler radar that operates at the frequency of 24.1 GHz. It provides DSD information by converting measured Doppler spectra into drop diameters by a known relationship. Parameters such as Rain rates (R), Liquid water content (LWC), falling velocity (W) and radar reflectivity (Z) were calculated from the Drop size distribution as measured directly by the Micro rain radar.

According to Das et. al., (2010), the spectral volume reflectivity $\eta(f)$ received by the radar with depth $\partial r$ is given by:

$$
\begin{equation*}
\eta(r, f) d f=p(r, f) d f . c \frac{r^{2}}{\partial r} t^{-1}(r) \tag{3}
\end{equation*}
$$

where $p(r, f)$ is the spectral power, f is the Doppler frequency in Hz . C is the radar constant. The DSD is calculated from the volume reflectivity $\eta(D)$ related thus:

$$
\begin{equation*}
N(D)=\frac{\eta(D)}{\sigma D} \tag{4}
\end{equation*}
$$

$N(D)$ is the number of drops with size D to $\mathrm{D}+\Delta \mathrm{D}$ in $\mathrm{m} / \mathrm{s}$.

The mean fall velocity $\left(\mathrm{V}_{\mathrm{m}}\right)$ is given by:

$$
\begin{equation*}
V_{m}=\frac{\lambda}{2} \int_{0}^{\infty} f \cdot P(f) d f \quad \square \tag{5}
\end{equation*}
$$

$P(f)$ is the spectral power related to Doppler frequency $\lambda$ is the wavelength.

The Micro rain radar is a very unique equipment that measures quantitative value of rain rate, drop size distributions, radar reflectivity, fall velocity, liquid water content and some other parameters simultaneously in vertical profiles from height of 0.16 to 4.8 km above the radar. It operates with electromagnetic radiation at a frequency of 24 GHz with modulation ranging from $0.5-0.15 \mathrm{MHz}$ depending on the height resolution.


Figure 1: Map of The Federal University of Technology Akure indicating Physics Department where the measurements were taken.

## 3. Modelling of DSD

The DSD is well represented by an expression developed by Marshall and Palmer (1948) and found out that it follows a function of the form:

$$
\begin{equation*}
N(D)=N_{O} \exp (-\Lambda D) \tag{5}
\end{equation*}
$$

where $N(D)$ is the concentration of raindrops per diameter in the diameter interval dD in mm, D is the rain drop diameter, $N_{o}$ is the intercept parameter with fixed value of $8 \times 10^{3} \mathrm{~mm}^{-1} \mathrm{~m}^{-3}$, $\wedge\left(\mathrm{mm}^{-1}\right)$ is the slope parameter and is defined as,

$$
\begin{equation*}
\wedge=4.1 \mathrm{R}^{-0.21} \mathrm{~mm}^{-1} \tag{6}
\end{equation*}
$$

where $R$ is the rainfall rate ( $\mathrm{mm} / \mathrm{h}$ ).
Marshall-Palmer discovered that rain DSDs' for several rain rates, the exponential function does not fit the observation. Hence, it is sometimes necessary to consider the Marshall Palmer curves applicable at diameters greater than 1-1.5mm. (Battan, 1973).

Log-normal distributions are usually characterized in terms of the log-transformed variable using as parameters, the expected values, or means, of its distribution and the standard deviation. The log-normal distributions are symmetrical also at the log level (Eckhard et al., 2001).

Log-normal representation is suitable for a broad range of applications and can facilitate interpretation of the physical processes that control the shape of the distribution.

Mahen et al., (2006) expresses Lognormal distribution as:

$$
\begin{equation*}
N(D)=\frac{N_{t}}{(2 \pi)^{0.5} \ln D} \exp \left[-\frac{\ln ^{2}\left(D / D_{g}\right)}{2 \ln ^{2} \delta}\right] \tag{7}
\end{equation*}
$$

where $N_{t}$ is the total number of drops per unit volume, $D_{g}$ is the geometric mean of the drop diameter in $\mathrm{mm}, \delta$ is the standard deviation of D .

Expression for the lognormal from equation (2.7) is reproduced as:

$$
\begin{equation*}
N(D)=(\exp A / D) \exp \left\{-0.5[(\ln D-B) / C]^{2}\right\} \tag{8}
\end{equation*}
$$

where,

$$
\begin{equation*}
A=\ln \left[\frac{N_{T}}{\sqrt{(2 \pi) \ln \sigma}}\right] \tag{9}
\end{equation*}
$$

$B=\ln D_{g}$
$C=\ln \sigma$
A, B and C in equations (2.10) to (2.12) are fit parameters of the lognormal distribution.
$\ln D_{g}$ and $\ln \sigma$ are values of geometric mean of drop diameters and standard deviation of drop diameters respectively. They are both calculated by:

$$
\begin{align*}
& \ln D_{g}=\left(\frac{1}{N_{T}}\right) \sum_{i=1}^{20} N_{i} \ln D_{i}  \tag{12}\\
& \ln ^{2} \sigma=\left(\frac{1}{N_{T}}\right) \sum_{i=1}^{20} N_{i}\left(\ln D_{i}-\ln D_{g}\right)^{2} \tag{13}
\end{align*}
$$

$N_{T}$ is the drop number concentration $\left(\mathrm{m}^{-3}\right)$ in the observed spectrum and $N_{i}$ is the number of drops in size category $D_{i}$ (Jassal et al., 2011).

## 4. Results

### 4.1 Modelling of DSD



Figure 2: Comparison of measured and model rain dropsize distribution during stratiform rainfall types


Figure 3: Comparison of measured and model rain dropsize distribution during convective rainfall types

Applying equation (6) to equation (8), we obtained the rainfall DSD for various types as given in Figures 2 and 3 for all the rainfall types for years 2008, 2009 and 2010. For each of
the statistical models, the input parameters closer to the southwestern region were considered as a typical case of a tropical situation. The figures show that the DSD results for each of the year indicate that the drop sizes measured varies from 0.25 mm in diameter to about 4.80 mm , with the larger concentration of the diameter around $0.25-0.56 \mathrm{~mm}$. As the rain drop diameter increases the drop size concentration decreases. This is in agreement with the work of Tokay et al. (1995).

In the modeling of DSD, comparison between lognormal and Marshall and Palmer (M-P) distribution are as shown in Figures 3 and 4. This represents an exponential fit of equation (6) for the drop size distribution for the actual average rain rates for stratiform and convective rain types in $\mathrm{mm} / \mathrm{hr}$. For stratiform rain type, the flattering of the measured distributions from the plots show that it is quantitatively in agreement with the M - P based distribution for all the rainfall types and deviate sharply from the lognormal models.

### 4.2 Yearly Cumulative Distribution of Rain Rates



Figure 4: Yearly cumulative distribution of rain rate


Figure 5: Average cumulative distribution of rain rate with ITU

The cumulative distribution of rain rates and the number of occurrence of rain events during the rainy periods were determined for some categories of rain rates.

Figures 4 and 5 are the plots of rain rates versus percentage of time for years 2008, 2009 and 2010. The result shows that Akure with an average annual rainfall accumulation of 1599 mm recorded about 78,74 and $81 \mathrm{~mm} / \mathrm{hr}$ at $0.01 \%$ of time in the first, second and third year respectively as shown in Figure. Year 2010 recorded more rain than that of 2008 and 2009 considered in this work and this shows a dynamic pattern of rain rate over the location.

The cumulative distribution of measured rain rate compared with ITU rain model is also presented in Figure 5. Results of the plot indicate that rain rate of high values correspond to lower percentage of time while the lower rainfall rate has higher percentage of time. It could further be observed that the recent ITU-R P.837-S (2007) model underestimated the rain rate values in this region. For example, at $0.01 \%$ of time the measured rain rate was about $35 \%$ under estimated by the ITU-R model.

## 4.3: Rain Microstructure



Figure 6: Plots of (a)Average fall velocity (b) Average rain rates (c) Average liquid water content (d) Absolute distribution of rain rates for 27th May 2014


Figure 7: Plots of (a)Average fall velocity (b) Average rain rates (c) Average liquid water content (d) Absolute distribution of rain rates for 23rd June 2014

| Range of Rain Rates (mm/hr) | ABSOLUTE DISTRIBUTION OF RAIN RATES |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | $27^{\text {th }}$ May, 2014 | $23^{\text {rd }} \text { June, } 2014$ | ${ }_{2014}{ }^{\text {th }} \text { October, } 2014$ | $5^{\text {th }}$ November, |
| 0-5 | 136 | 294 | 29 | 167 |
| 5-10 | 16 | 35 | 5 | 8 |
| 10-15 | 6 | 10 | 6 | 4 |
| 15-20 | 7 | 6 | 3 | 2 |
| 20-25 | 4 | 2 | 1 | 0 |
| 25-30 | 2 | 1 | 1 | 1 |
| 30-35 | 2 | 0 | 0 | 0 |
| 35-40 | 2 | 0 | 2 | 0 |
| 40-45 | 1 | 0 | 0 | 0 |
| 45-50 | 0 | 0 | 0 | 0 |
| 50-60 | 2 | 0 | 1 | 0 |
| 60-70 | 1 | 0 | 0 | 0 |
| 70-80 | 1 | 0 | 0 | 0 |
| >100 | 1 | 0 | 0 | 1 |

Table 1: Absolute Distribution of Rainfall for Year 2014

The average fall velocities, rain rate, liquid water content and the absolute distribution of rain rate for some rainy periods in year 2014 were plotted using the METEK graphics (Figures 4.43 to 4.46 ) and the summary of the results of the absolute distribution of rain rates are as shown in Table 1.

Results obtained indicate that on the $27^{\text {th }}$ of May 2014, rain started from about $02: 15$ to 03:00am and the highest rain rate of $119.49 \mathrm{~mm} / \mathrm{hr}$ occurred at $02: 19 \mathrm{hr}$ and the highest LWC is $26.14 \mathrm{~g} / \mathrm{m} 3$ also at this hour. About $84.98 \%$ of the total rainfall is stratiform while $15.02 \%$ is convective. Also, on the $23^{\text {rd }}$ of June 2014, $94.54 \%$ of the total rainfall was stratiform while $5.46 \%$ was convective. It is important to note that for $30^{\text {th }}$ October 2014 an high rate of convective rain events are recorded and precisely we have $70.1 \%$ of the total rainfall as stratiform while $29.9 \%$ is convective and on the $5^{\text {th }}$ November of the same year $95.63 \%$ of the total rainfall is stratiform while $4.37 \%$ of the rain is convective.

### 5.0 Conclusion

Rain events for years 2008, 2009, 2010 and 2014 collected using a Micro Rain Radar were used for this research. They were classified into high and low intensity rains according to the values of rain rates. The high intensity rains were further classified into shower and thunderstorm, while the low intensity rain were classified into drizzle and widespread. The

DSD results show that the drop diameter increases as the drop-size concentration decreases. From the plot of the yearly cumulative distribution of rain rate for years 2008, 2009 and 2010, it was observed that year 2010 recorded more rain than the other two years and the percentage of time decreases as the rain rate increases.

It was also observed that most of the rain events in this part of the world is the low intensity rain (stratiform) i.e. rain rates below $10 \mathrm{~mm} / \mathrm{hr}$. So, it can be concluded that over $85 \%$ of the total rainfall in this part of the world is stratiform while the remaining $15 \%$ are convective except for the month of October which is the peak of the rainy season in the year where a high number of convective rain is observed. This in essence means that convective rain only fall for a short period of time while stratiform rain fall for a long period of time.

The plots also show typical values for negative exponential rainfall DSD similar to that of Marshall-Palmer (1948) which is also in good agreement with the work of Adimula (1997) for stratiform rain type. The results of the rainfall DSD for the four rainfall types used in this study at different rainfall rates: $4.99 \mathrm{~mm} / \mathrm{hr}$ (for drizzle), $7.5 \mathrm{~mm} / \mathrm{hr}$ (for widespread), 25 $\mathrm{mm} / \mathrm{hr}$ (for shower) and $110 \mathrm{~mm} / \mathrm{hr}$ (for thunderstorm) were also presented. These rainfall rates are selected based on the spectral characteristic within their respective rainfall types as they can however provide an insight into the behaviour of the existing statistical models. Rain drops in the diameter bin of 0.25 mm which represent the drop spectrum N04 contributed most to the rain fall event throughout this period over each of the rain types.

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